哈尔滨工业大学深圳研究生院

2012年 秋 季学期期末考试试卷

HIT Shenzhen Graduate School Examination Paper

**Course Name:**  组合数学  **Lecturer:** 黄荷姣

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| **Question** | **One** | **Two** | **Three** | **Four** | **Five** | **Six** | **Seven** | **Eight** | **Nine** | **Ten** | **Total** |
| **Mark** |  |  |  |  |  |  |  |  |  |  |  |

The following formula and results may be useful for you in this examination.



The number of ways to place n non-attacking, indistinguishable rooks on an n-by-n board with forbidden positions equals n! – r1(n-1)! + r2(n-2)! - ... + (-1)k rk(n-k)!+…+(-1)rrn.

For a specified grid *i* in chessboard C, let Ci be a chessboard induced from C by deleting the row and the column of grid *i*；let Ce be induced from C by deleting grid *i* from C. Then, rk(C)=rk－1(Ci)＋rk(Ce)，R(C)= xR(Ci) + R(Ce).

Assume that the sequence h0, h1, h2, …, hn,… has a difference table whose 0th diagonal equals, c0, c1, c2, …, cp≠0, 0, 0, … Then, hn = c0 C(n, 0) + c1C(n, 1) + c2C(n, 2) + … +cpC(n, p). and



Let n ≥ 2 be an integer. If there exist n-1 MOLS of order n, then there exists a resolvable BIBD with parameters b = n2+n, v = n2, k = n, r = n+1, = 1.





, where type(f) = (e1, e2,…,en).

Question One: Please give the answers for the following 18 questions **without explanation.**

1. (3’) A football team of 11 players is to be selected from a set of players, 5 of whom can play only in the backﬁeld, 8 of them can only play on the line, 2 of them can play either in the backﬁeld, or on the line. Assuming a football team has 7 men on the line and 4 men in the backﬁeld, the number of possible football teams is ( C(8,7)C(7,4)+C(2,1)C(8,6)C(6,4)+ C(8,5)C(5,4) ).
2. (3’) Determine an integer mn such that if mn points are chosen within an equilateral triangle of side length 1, there are two whose distance apart is at most . mn = ( (n+1)n )
3. (3’) Consider the permutation of {1,2,3,4,5,6,7,8}. The inversion sequence of 83476215 is ( 65113210 ); The permutation with an inversion sequence 66142100 is ( 73658421 ).
4. (3’) In which position does the combination 1289 occur in the lexicographic order of the 4-combinations of {1, 2, 3, 4, 5, 6, 7, 8, 9}? ( C(9,4)-C(8,4)-C(7,3) ).
5. (3’) Among the combination of {x7, x6, …, x1, x0}, what is the combination that immediately follow {x7, x5, x4, x3, x2, x1, x0} by using the base 2 arithmetic generating scheme? ( {x7, x6})
6. (3’) In how many ways can six women, eight men and a dog sit around a table such that no two women sit next to each other? 8!P(9,6)
7. (3’) How many circular permutations are there of the multiset {3·a, 4·b, 2·c, 1·d}, where for each type of letters, all letters of that type do not appear consecutively.

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1. (3’) Let equal the number of different ways in which the squares of a 1-by-n chessboard can be colored, using the colors red, green, and black so that no two squares that are colored red are adjacent. Then, the recurrence relation foris ( ).

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1. (3’) Determine the generating function for the number hn of non-negative integral solution of .
2. (3’) Let equal the number of different ways in which the squares of a 1-by-n chessboard can be colored, using the colors red, blue, white, and green, so that the number of squares colored red is even, and the number of squares colored white is odd. Then = ( ).
3. (3’) Let the sequence be defined by .

Then = ( 3C(n+1,1)+ C(n+1,2)+ 4C(n+1,3) ).

1. (3’) Let

f =  and g = 

Let c=(R, B, B, R, R, R) be a coloring of 1, 2, 3, 4, 5, 6 with the colors R and B. Then (g) and (f)

1. (3’) Suppose 20 indistinguishable balls are allocated to 5 persons so that everyone has no less than 3 balls. Then the number of ways is ( ). If the balls are distinguishable, then the number of ways is ( ).
2. (3’) In how many ways can 5 red rooks and 3 blue rooks can be placed on 10×10 board so that no two rooks can attack one another, where the first line and the first column should not be empty.

C(9,7) C(9,7)8!8!/5!5!3!3!

1. (3’) There is a party with 4 couples. In how many ways can they choose their partner, where each pair of partners is not couples?

D4=4!-C(4,1)3!+C(4,2)2!- C(4,3)1!+C(4,4)0!

1. (3’) Count the permutations of where ; , ,
2. (3’) Apply the deferred acceptance algorithm to obtain a stable complete marriage for the following preferential ranking matrix

a b c d

A-c B-d C-a D-b

1. (3’) Complete the following semi-Latin square or order 5.

1 3 5 4 2

3 2 1 5 4

4 1 3 2 5

2 5 4 1 3

5 4 2 3 1

**Note: Please give a detailed explanation for the following Five problems.**

Question One (6’): Prove that, among any 50 points in a square of size 7cm × 7cm, there are two points whose distance apart is at mostcm.

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We divide the square into 49 portions .In each square ,the longest distance of two points is cm.

According to Pigeonhole principle ,there must be two points ,which are in the 50 point ,in the same square .

So there are two points whose distance apart is at mostcm.

Question Two (10’): Let ，where , , , , , . Does A has SDRs ? Why?What is the largest number of sets of A which can be chosen such that they have an SDR?

No, it hasn't .

|A|=6

(1)When k=1

i=1,2...6

min |Ai|+6-k=6

(2)when k=2

i ,j=1,2...6

min |Ai∪Aj|+6-k=6

(3)when k=3

i ,j,k=1,2...6

min |Ai∪Aj∪Ak|+6-k=6

(4)when k=4

i ,j,k,l=1,2...6

min |Ai∪Aj∪Ak∪Al|+6-k=5

(5)when k=5

i ,j,k,m,l=1,2...6

min |Ai∪Aj∪Ak∪Al∪Am |+6-k=5

(6)when k=6

i ,j,k,m,l,n=1,2...6

min |Ai∪Aj∪Ak∪Al∪Am∪An |+6-k=5

The largest number of A , which can be chosen such that they have an SDR ,is 5 .

Question Three (10’): Let p be a prime number. Determine the number of different necklaces that can be made from p beads of n different colors. Specially, how many different necklaces that are made from 5 beads of 2 red, 1 yellow and 2 white colors?

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=0

=0

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=0

N(,C)=p(p!+0+0+...)=(p-1)!

When p=5:

N(,C)=!

Question Four (10’): Suppose there are 16 varieties of products which need to be tested by 20 consumers. The test is to have a property that each pair of the 16 varieties is compared by exactly one person. Each consumer is responsible for 4 products and each product is tested 5 times. Please give a proper solution.

Answer:

Let b: the number of consumers.

v: the number of varieties.

k: the number of varieties tested by each consumer.

r: the number of consumers containing each variety.

: the number of consumers containing each pair of varieties.

So b=20, v=16, k=4, r=5, =1. To construct a prober solution with parameters:



n=4.

Let  denote three MOLS of order four.

  

The varieties are 5 positions of a 5-by-5 array, and the result is pictured as follows:



Hence the proper solution is:



Question Five (10’): Determine the number of integral solutions of the following equation

x1+x2+…+x5= 18

which satisfy 0 ≤ x1 ≤ 3, 3 ≤ x2 ≤ 10, 2 ≤ x3 ≤ 10, 1 ≤ x4 ≤ 3, 5 ≤ x5 ≤ 8.

Answer:

We introduce new variables



And our equation becomes



The inequalities on the  are satisfied if and only if



Let S be the set of all nonnegative integral solutions of ,the size of S is



Let  be the property that , be the property that , be the property that , be the property that , be the property that .Let denote the subset of S consisting of the solutions satisfying property ,(i=1,2,3,4,5).We wish to evaluate the size of the set ,and we do so by applying the inclusion-exclusion principle. The set A1 consist of all those solutions in S for which .Performing a change in variable (), we see that the number of solutions in A1 is the same as the number of nonnegative integral solutions of



Hence,



In a similar way, we obtain

,,,

The set  consists of all those solutions in S for which and .

In a similar way, we obtain



Hence,

